RS Aggarwal Solutions Class 10 Chapter 1 Real Numbers

Exercise 1A

Questions 1:

For any two given positive integers a and b there exist unique whole numbers q and r such that
Here, we call 'a' as dividend, b as divisor, q as quotient and r as remainder.
Dividend = (divisor quotient) + remainder

Questions 2:

By Euclid's Division algorithm we have:
Dividend = (divisor * quotient) + remainder
= (61 * 27) + 32 = 1647 + 32 = 1679

Questions 3:

By Euclid's Division Algorithm, we have:
Dividend = (divisor quotient) + remainder

$$1365 = (\text{divisor} \times 31) + 32$$
$$\frac{1365 - 32}{31} = \text{divisor}$$
$$\Rightarrow \frac{1331}{31} = \text{divisor}$$
$$\therefore \text{Divisor} = 43$$

Questions 4:
(i) On dividing 2520 by 405, we get
Quotient = 6, remainder = 90
2520 = (405 x 6) + 90
Dividing 405 by 90, we get Quotient = 4, Remainder = 45
405 = 90 x 4 + 45
Dividing 90 by 45 We get Quotient = 2, remainder = 0
90 = 45 x 2
H.C.F. of 405 and 2520 is 45

```
405)2520 ( 6
    2430
    ----
     90) 405 (4
         360
         ---
         45)90(2
            90
            --
             ×
```

(ii) Dividing 1188 by 504, we get Quotient = 2, remainder = 180

1188 = 504 x 2+ 180

Dividing 504 by 180 Quotient = 2, remainder = 144

504 = 180 x 2 + 144

Dividing 180 by 144, we get Quotient = 1, remainder = 36

Dividing 144 by 36

Quotient = 4, remainder = 0

H.C.F. of 1188 and 504 is 36

```
504)1188 ( 2
    1008
    ----
    180)  504 (2
          360
          ---
          144) 180 (1
               144
               ---
               36)144( 4
                  144
                  ---
                   ×
```

(iii) Dividing 1575 by 960, we get

Quotient = 1, remainder = 615

1575 = 960 x 1 + 615

Dividing 960 by 615, we get Quotient = 1, remainder = 345

960 = 615 x 1 + 345

Dividing 615 by 345 Quotient = 1, remainder = 270

615 = 345 x 1 + 270

Dividing 345 by 270, we get Quotient = 1, remainder = 75

345 = 270 x 1 + 75

Dividing 270 by 75, we get Quotient = 3, remainder =45

270 = 75 x 3 + 45

Dividing 75 by 45, we get Quotient = 1, remainder = 30

75 = 45 x 1 + 30

Dividing 45 by 30, we get Remainder = 15, quotient = 1

45 = 30 x 1 + 15

Dividing 30 by 15, we get Quotient = 2, remainder = 0

H.C.F. of 1575 and 960 is 15

```
960)1575 ( 1
     960
    615)  960 (1
          615
          345) 615 (1
               345
               270)345( 1
                   270
                     75)270 (3
                        225
                          45)75(1
                             45
                              30) 45(1
                                  30
                                  15) 30(2
                                      30
                                       ×
```

Questions 5:

(i) By prime factorization, we get

2	144
2	72
2	36
2	18
3	9
3	3
	1

2	198
3	99
3	33
11	11
	1

$$\therefore 144 = 2\times2\times2\times2\times3\times3 = 2^4\times3^2$$

$$\therefore 198 = 2\times3^2\times11$$

$$\therefore \text{H.C.F. of } (144,198) = 2\times3^2 = 2\times3\times3 = 18$$

$$\text{L.C.M of 144 and 198} = 2^4\times3^2 = 11$$

$$= 16\times9\times11 = 1584$$

(ii) By prime factorization. We get

2	396
2	198
3	99
3	33
11	11
	1

2	1080
2	540
2	270
3	135
3	45
3	15
5	5
	1

$\therefore 396 = 2^2 \times 3^2 \times 11$

$\therefore 1080 = 2^3 \times 3^3 \times 5$

$\therefore$ H.C.F. of $(396, 1080) = 2^2 \times 3^2 = 4 \times 9 = 36$

L.C.M of 396 and 1080 $= 2^3 \times 3^3 \times 5 \times 11 = 11880$

(iii) By prime factorization, we get

2	1152
2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

2	1664
2	832
2	416
2	208
2	104
2	52
2	26
13	13
	1

$\therefore 1152 = 2^7 \times 3^2$

$\therefore 1664 = 2^7 \times 13$

$\therefore$ H.C.F. of $(1152, 1664) = 2^7 = 128$

L.C.M of 1152 and 1664 $= (2^3 \times 3^3 \times 13) = 128 \times 9 \times 13 = 14976$

Questions 6:

(i) By prime factorization, we get

2	24
2	12
2	6
3	3
	1

2	36
2	18
3	9
3	3
	1

2	40
2	20
2	10
5	5
	1

$\therefore 24 = 2^3 \times 3$

$36 = 2^2 \times 3^2$

$40 = 2^3 \times 5$

$\therefore$ H.C.F. of $(24, 36, 40) = 2^2 = 4$

L.C.M of 24, 36 and 40 $= \left(2^3 \times 3^3 \times 5\right) = (8 \times 9 \times 5) = 360$

(ii) By prime factorization, we get

2	30
3	15
5	5
	1

2	72
2	36
2	18
3	9
3	3
	1

2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

$\therefore 30 = 2 \times 3 \times 5$

$72 = 2^3 \times 3^2$

$432 = 2^4 \times 3^3$

$\therefore$ H.C.F. of $(30, 72, 432) = 2 \times 3 = 6$

L.C.M of 30, 72 and 432 $= \left(2^4 \times 3^3 \times 5\right)$

$= (16 \times 27 \times 5)$

$= 27 \times 80 = 2160$

(iii) By prime factorization, we get

3	21
7	7
	1

2	28
2	14
7	7
	1

2	36
2	18
3	9
3	3
	1

3	45
3	15
5	5
	1

$\therefore 1152 = 2^7 \times 3^2$

$\therefore 1664 = 2^7 \times 13$

$\therefore$ H.C.F. of $(1152, 1664) = 2^7 = 128$

L.C.M of 1152 and 1664 $= (2^3 \times 3^3 \times 13) = 128 \times 9 \times 13 = 14976$

Questions 7:

H.C.F = 23; L.C.M, = 1449

For any two numbers a and b, we have

$a \times b = L.C.M \times H.C.F$

$$\therefore b = \frac{L.C.M \times H.C.F}{a}$$

$$\Rightarrow b = \frac{1449 \times 23}{161} = 207$$

Questions 8:

H.C.F. of two numbers = 11, their L.C.M = 7700

One number = 275, let the other number be b

Now, 275 x b = 11 x 7700

$$\therefore b = \frac{11 \times 7700}{275} = 308$$

Questions 9:

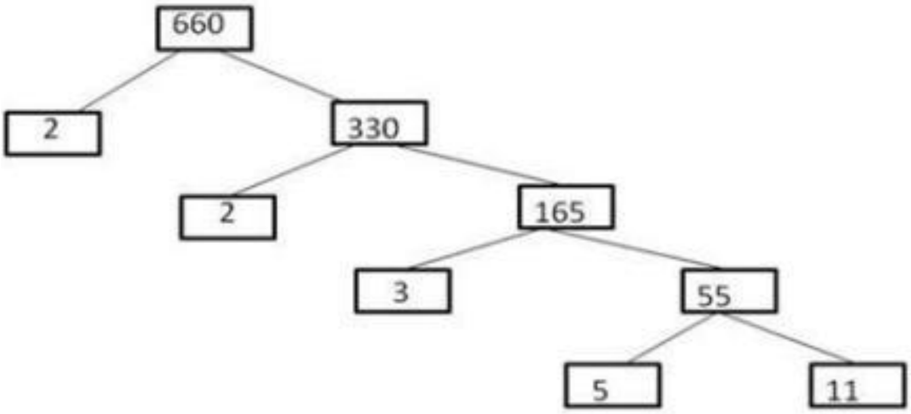
By going upward

5 x 11= 55

55 x 3 = 165

165 x 2 = 330

330 x 2 = 660



Questions 10:

Subtracting 6 from each number:

378 – 6 = 372, 510 – 6 = 504

Let us now find the HCF of 372 and 504 through prime factorization:

372 = 2 x 2 x 3 x 31

2	372
2	186
3	93
	31

2	504
2	252
2	126
7	63
3	9
	3

504 = 2 x 2 x 2 x 3 x 3 x 7

$= 2^2 \times 3 \times 31$

The required number is 12.

$= 2^3 \times 3^2 \times 7$

$\text{H.C.F of } 372 \text{ and } 504 = 2^2 \times 3 = 12$

Questions 11:

Subtracting 5 and 7 from 320 and 457 respectively:

320 – 5 = 315,

457 – 7 = 450

Let us now find the HCF of 315 and 405 through prime factorization:

3	315
3	105
5	35
	7

2	450
3	225
3	75
5	25
	5

$315 = 3 \times 3 \times 5 \times 7$

$\quad = 3^2 \times 5 \times 7$

$450 = 2 \times 3 \times 3 \times 5 \times 5$

$\quad = 2 \times 3^2 \times 5^2$

The required number is 45.

Questions 12:

(i) $\frac{69}{92} = \frac{3 \times 23}{2 \times 2 \times 23} = \frac{3}{4}$

3	69
	23

2	92
2	46
	23

(ii) $\frac{561}{748} = \frac{3 \times 11 \times 17}{2 \times 2 \times 11 \times 17} = \frac{3}{4}$

3	561
11	187
	17

2	748
2	374
11	187
	17

(iii) $\frac{1695}{1168} = \frac{3 \times 5 \times 73}{2 \times 2 \times 2 \times 2 \times 73} = \frac{15}{16}$

3	1095
5	365
	73

2	1168
2	584
2	292
2	146
	73

Questions 13:

The prime factorization of 42, 49 and 63 are:

42 = 2 x 3 x 7, 49 = 7 x 7, 63 = 3 x 3 x 7

Therefore, H.C.F. of 42, 49, 63 is 7

Hence, greatest possible length of each plank = 7 m

Questions 14:

7 m = 700cm, 3m 85cm = 385 cm

12 m 95 cm = 1295 cm

Let us find the prime factorization of 700, 385 and 1295:

2	700
2	350
5	175
5	35
	7

5	385
7	77
	11

5	1295
7	259
	37

Now, $700 = 2 \times 2 \times 5 \times 5 \times 7 = 2^2 \times 5^2 \times 7$

$385 = 5 \times 7 \times 11$

$1295 = 5 \times 7 \times 37$

$\therefore$ H.C.F. $= 5 \times 7 = 35$cm

Greatest possible length = 35cm.

Questions 15:

Let us find the prime factorization of 1001 and 910:

1001 = 11 x 7 x 13

910 = 2 x 5 x 7 x 13

11	1001
7	91
	13

2	910
5	455
7	91
	13

H.C.F. of 1001 and 910 is 7 x 13 = 91

Maximum number of students = 91

Questions 16:

Let us find the HCF of 336, 240 and 96 through prime factorization:

2	336
2	168
2	84
2	42
3	21
	7

2	240
2	120
2	60
2	30
3	15
	5

2	96
2	48
2	24
2	12
2	6
	3

$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7 = 2^4 \times 3 \times 7$

$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5 = 2^4 \times 3 \times 5$

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3$

$HCF = 2^4 \times 3 = 16 \times 3 = 48$

Each stack of book will contain 48 books

Number of stacks of the same height

$$= \frac{240}{48} + \frac{336}{48} + \frac{96}{48} = 5 + 7 + 2 = 14$$

Questions 17:

Length of ceiling = 15m 17cm = 1517 cm

Its breadth = 9m 2cm = 902 cm

```
902)1517 ( 1
     902
    615)  902 (1
          615
          287) 615 (2
                574
                 41)287( 7
                    287
                     ×
```

$\therefore$ H.C.F. of 1517 and 902 = 41

Maximum size of tile = 41cm × 41cm

$$\text{Least number of tiles} = \frac{1517 \times 902}{41 \times 41} = 37 \times 22 = 814$$

Questions 18:

Let us find the LCM of 64, 80 and 96 through prime factorization:

2	64
2	32
2	16
2	8
2	4
	2

2	80
2	40
2	20
2	10
	5

2	96
2	48
2	24
2	12
2	6
	3

$$64 = 2\times2\times2\times2\times2\times2 = 2^6$$

$$80 = 2\times2\times2\times2\times5 = 2^4\times5$$

$$96 = 2\times2\times2\times2\times2\times3 = 2^5\times3$$

L.C.M of 64, 80 and 96 =

$$= 2^6 \times 5 \times 3 = 64 \times 15 = 960\text{cm} = 9.6\text{m}$$

Therefore, the least length of the cloth that can be measured an exact number of times by the rods of 64cm, 80cm and 96cm = 9.6m

Questions 19:

Let us find the LCM of 48, 72 and 108 through prime factorization:

2	48
2	24
2	12
2	6
	3

2	72
2	36
2	18
3	9
	3

2	108
2	54
3	27
3	9
	3

$48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$

$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$

$108 = 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3$

$\therefore \text{LCM of } 48, 72, 108 \text{ is } 2^4 \times 3^3$

$= 16 \times 27 \text{ sec} = 432 \text{ sec} = 7 \text{ min } 12 \text{ sec}$

Three bells toll together after 7 min 12 sec

Questions 20:

Interval of beeping together = LCM (60 seconds, 62 seconds)

The prime factorization of 60 and 62:

60 = 30 x 2, 62 = 31 x 2

L.C.M of 60 and 62 is 30 x 31 x 2 = 1860 sec = 31min

electronic device will beep after every 31minutes

After 10 a.m., it will beep at 10 hrs 31 minutes

Questions 21:

$2 = 2 \times 1$

$4 = 2 \times 2$

$6 = 2 \times 3$

$8 = 2 \times 2 \times 2$

$10 = 2 \times 5$

$12 = 2 \times 2 \times 3$

L.C.M of 2, 4, 6, 8, 10, 12 minutes

$= 2 \times 2 \times 2 \times 3 \times 5 = 120$ minutes = 2 hours

After every 2 hours they toll together.

Required number of times = $\left(\frac{30}{2} + 1\right) = 16$ times

Exercise 1B

Questions 1:

(i) $\frac{11}{2^3 \times 3}$

Its denominator, $\left(2^3 \times 3\right) \neq \left(2^m \times 5^n\right)$

$\therefore$ it is a non – terminating repeating decimal

(ii) $\frac{73}{2^2 \times 3^3 \times 5}$

Its denominator, $\left(2^2 \times 3^3 \times 5\right) \neq \left(2^m \times 5^n\right)$

$\therefore$ it is a non – terminating repeating decimal

(iii) $\frac{9}{35} = \frac{9}{5 \times 7}$

Its denominator, $(5 \times 7) \neq \left(2^m \times 5^n\right)$

$\therefore$ it is a non terminating repeating decimal

(iv) $\frac{32}{147} = \frac{32}{7 \times 3 \times 7} = \frac{32}{3 \times 7^2}$

Its denominator, $\left(3 \times 7^2\right) \neq \left(2^m \times 5^n\right)$

$\therefore$ it is a non- terminating repeating decimal

(v) $\frac{64}{455} = \frac{64}{5 \times 7 \times 13}$

Its denominator, $(5 \times 7 \times 11) \neq \left(2^m \times 5^n\right)$

$\therefore$ it is a non – terminating repeating decimal

(vi) $\frac{77}{210} = \frac{77}{3 \times 2 \times 5 \times 7}$

Its denominator, $(2 \times 3 \times 5 \times 7) \neq \left(2^m \times 5^n\right)$

$\therefore$ it is a non – terminating repeating decimal

(vii) $\frac{29}{343} = \frac{29}{7 \times 7 \times 7} = \frac{29}{7^3}$

Its denominator, $\left(7^3\right) \neq \left(2^m \times 5^n\right)$

$\therefore$ it is a non – terminating repeating decimal

(viii) $\frac{129}{2^2 \times 5^7 \times 7^5}$

Its denominator, $\left(2^2 \times 5^7 \times 7^5\right) \neq \left(2^m \times 5^n\right)$

$\therefore$ it is a non – terminating repeating decimal

Questions 2:

(i) $\frac{23}{2^3 \times 5^2}$ has 2 and 5 as factors in denominator

$$\frac{23}{2^3 \times 5^2} = \frac{23}{2 \times 100} = \frac{11.5}{100} = 0.115$$

(ii) $\frac{24}{125} = \frac{24}{5^3}$ has 5 as its factor in denominator

$$\frac{24 \times 8}{125 \times 8} = \frac{192}{1000} = 0.192$$

(iii) $\frac{170}{320} = \frac{17}{2^6 \times 5}$ has 2 and 5 as factors in the denominator.

$$\frac{17}{32 \times 10} = \frac{17 \times 5^5}{2^5 \times 5^5 \times 10} = \frac{53125}{10^5 \times 10} = 0.053125$$

(iv) $\frac{171}{800} = \frac{171}{2^5 \times 5^2}$ has 2 and 5 as its factors in the denominator

$$\frac{171}{2^5 \times 5^2} = \frac{171 \times 5^3}{2^5 \times 5^2 \times 5^3} = \frac{21375}{10^5} = 0.21375$$

(v) $\frac{15}{1600} = \frac{15}{2^6 \times 5^2}$ has 2 and 5 as its factors in the denominator

$$\frac{15}{2^6 \times 5^2} = \frac{15 \times 5^4}{2^6 \times 5^2 \times 5^4} = \frac{9375}{10^6} = 0.0099375$$

(vi) $\frac{19}{3125} = \frac{19}{5^5}$ has 5 as its factors denominator

$$\frac{19}{5^5} = \frac{19 \times 2^5}{5^5 \times 2^5} = 0.00608$$

Questions 3:

(i) $x = 0.\bar{8}$ then,

$x = 0.8888......$ ---------(1)

$\therefore 10x = 8.8888.....$ ----------(2)

On subtracting (1) from (2), we get

$9x = 8 \Rightarrow x = \frac{8}{9}$

Hence, $0.\bar{8} = \frac{8}{9}$

(ii) $x = 2.\bar{4}$ then,

$x = 2.4444.....$ ---------(1)

$\therefore 10x = 24.444.....$ ---------(2)

On subtracting (1) from (2), we get

$9x = 22 \Rightarrow x = \frac{22}{9}$

(iii) $x = 0.\overline{24}$ then,

$x = 0.242424....$ -------(1)

$100x = 24.242424.....$ -----(2)

On subtracting (1) from (2), we get

$99x = 24 \Rightarrow x = \frac{24}{99} = \frac{8}{33}$

$0.\overline{24} = \frac{8}{33}$

(iv) $x = 0.1\bar{2}$, then

$x = 0.12222.....$

$10x = 1.2222.....$ -----(1)

$100x = 12.2222....$ -----(2)

Subtracting (1) from (2), we get

$90x = 11 \qquad \therefore x = \frac{11}{90}$

$0.1\bar{2} = \frac{11}{90}$

(v) $x = 2.2\overline{4}$, then,

$x = 2.24444$ ------(1)

$10x = 22.4444.....$ ------(2)

And $100x = 224.4444.....$ ------(3)

On subtracting (2) from (3), we get

$\therefore\ 90x = 202 \Rightarrow x = \frac{202}{90} = \frac{101}{45}$

(vi) $x = 0.3\overline{65}$, then

$x = 0.3656565$----(1)

$10x = 3.656565$ ----- (2)

$1000x = 365.656565$ ------ (3)

Subtracting (2) from (3), we get

$x = 362 \Rightarrow x = \frac{362}{990} = \frac{181}{495}$

$0.3\overline{65} = \frac{181}{495}$

Questions 4:

(i) 53.123456789 is a rational number since it is a terminating decimal.

(ii) $31.\overline{123456789}$ is a rational number because it is a non – terminating repeating decimal.

(iii) 0.12012001200012...... is not a rational number as it is a non-terminating, non – repeating decimal.

Exercise 1C

Questions 1:

<u>Rational Numbers</u>: The numbers of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ are called rational number.

<u>Irrational Numbers</u> : The numbers which when expressed in decimal form and expressible as non- terminating and non-repeating decimal are known as irrational number.

<u>Real numbers</u>: A number which is rational or irrational is called real number.

Questions 2:

(i) $\frac{22}{7}$ is rational

(ii) 3.1416 is a terminating and non-repeating decimal, so it is rational.

(iii) π is irrational

(iv) $3.\overline{142857}$ is in non-terminating repeating decimal form, so it is rational

(v) 5.636363...... is a non-terminating repeating decimal. So it is a rational.

(vi) 2.040040004.... is a non-terminating and non-repeating decimal, so it is irrational.

(vii) 0.535335333..... is a non-terminating and non-repeating decimal, so it is irrational.

(viii) 3.121221222..... is a non-terminating and non-repeating decimal. So it is an irrational number.

(ix) If 21 is a positive integer which is not a perfect square, then $\sqrt{21}$ is irrational.

(x) If 3 is a positive integer which is not a perfect cube, then $\sqrt[3]{3}$ is irrational.

Questions 3:

(i) If possible, let $\sqrt{6}$ be rational and let its simplest form be $\frac{a}{b}$ then, a and b are integers having no common factor other than 1, and $b \neq 0$.

Now, $\sqrt{6} = \frac{a}{b} \Rightarrow 6 = \frac{a^2}{b^2}$ [on squaring both sides]

$\Rightarrow \quad 6b^2 = a^2 \quad .....(1)$

$\Rightarrow 6$ divides a^2 $[\because 6 \text{ divides } 6b^2]$

$\Rightarrow 6$ divides a

Let a = 6c for some integer c

Putting a =6c in (1), we get

$6b^2 = 36c^2 \Rightarrow b^2 = 6c^2$

$\Rightarrow 6$ divides b^2 $[\because 6 \text{ divides } 6c^2]$

$\Rightarrow 6$ divides b $[\because 6 \text{ divides } b^2 = 6 \text{ divides } b]$

Thus, 6 is a common factor of a and b

But, this contradicts the fact that a and b have no common factor other than 1

The contradiction arises by assuming that $\sqrt{6}$ is rational.

Hence $\sqrt{6}$ is irrational.

(ii) If possible let $2 - \sqrt{3}$ is rational

$\Rightarrow 2 - (2 - \sqrt{3})$ is rational

[$\because$ difference of two rationals is rational]

$\therefore \sqrt{3}$ is rational

This contradicts the fact $\sqrt{3}$ is irrational

Since the contradiction arises by assuming $2 - \sqrt{3}$ rational.

Hence, $2 - \sqrt{3}$ is irrational.

(iii) If possible let $3+\sqrt{2}$ is rational

$\Rightarrow \left(3+\sqrt{2}\right)-3=\sqrt{2}$ is rational

[$\because$ difference of two rational is rational]

$\therefore \sqrt{2}$ is rational

This contradicts the fact that $\sqrt{2}$ is irrational

Since the contradiction arises by assuming that $3+\sqrt{2}$ is rational.

Hence $3+\sqrt{2}$ is irrational.

(iv) If possible, let $2+\sqrt{5}$ is rational.

$\Rightarrow \left(2+\sqrt{5}\right)-2=\sqrt{5}$ is rational

[$\because$ difference of two rational is rational]

$\therefore \sqrt{5}$ is rational.

This contradicts the fact that $\sqrt{5}$ is irrational

Since, the contradiction arises by assuming $2+\sqrt{5}$ is rational.

Hence, $2+\sqrt{5}$ is irrational.

(v) If possible, let $5+3\sqrt{2}$ is rational

Now, $\left(5+3\sqrt{2}\right)-5=3\sqrt{2}$ is rational

[$\because$ Difference of two rational is rational]

Also, $\frac{1}{3}\times 3\sqrt{2}=\sqrt{2}$ is rational

[$\because$ Product of two rational is rational]

$\therefore \sqrt{2}$ is rational.

This contradicts the fact that $\sqrt{2}$ is irrational.

Since, the contradiction arises by assuming that $5+3\sqrt{2}$ is irrational.

Hence, $5+3\sqrt{2}$ is irrational

(vi) If possible, let $3\sqrt{7}$ be rational.

Let its simplest form be $3\sqrt{7} = \frac{a}{b}$, where a and b are positive integers having no common factor other than 1, then

$3\sqrt{7} = \frac{a}{b} \Rightarrow$

$\sqrt{7} = \frac{a}{3b} ------(2)$

Since, a and 3b are non –integers, so $\frac{a}{3b}$ is rational.

Thus, from (2), its follows that $\sqrt{7}$ is rational.

This contradicts the fact that $\sqrt{7}$ is irrational.

The contradiction arises by assuming that $3\sqrt{7}$ is rational.

Hence, $3\sqrt{7}$ is irrational.

(vii) $\frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3}{5}.\sqrt{5}$ -------(3)

If possible, let $\frac{3}{\sqrt{5}}$ be rational.

Then, from (3), it follows that $\frac{3}{5}.\sqrt{5}$ is rational

Let $\frac{3}{5}\sqrt{5} = \frac{a}{b}$, where a and b are non-zero integers having no common factor other than 1.

Now,

$\frac{3\sqrt{5}}{5} = \frac{a}{b} \Rightarrow$

$\sqrt{5} = \frac{5a}{3b}$ ------(4)

But, 3a and 5b are non-zero integers.

$\therefore \frac{5a}{3b}$ is rational.

Thus, from (4), it follows that $\sqrt{5}$ is rational.

This contradicts the fact that $\sqrt{5}$ is irrational.

The contradiction arises by assuming that $\frac{3}{\sqrt{5}}$ is rational.

Hence $\frac{3}{\sqrt{5}}$ is irrational.

(viii) If possible, let $2-3\sqrt{5}$ is rational.

$\Rightarrow \left(2-3\sqrt{5}\right)-2=-3\sqrt{5}$ is rational.

[$\because$ Difference of two rational is rational]

$\Rightarrow \left(-\frac{1}{3}\right)\times\left(-3\sqrt{5}\right)=\sqrt{5}$ is rational.

[$\because$ Product of two rationals is rational]

This contradicts that fact that $\sqrt{5}$ is irrational.

Since, the contradiction arises by assuming $2-3\sqrt{5}$ is rational.

Hence, $2-3\sqrt{5}$ is irrational.

(ix) If possible, let $\left(\sqrt{3}+\sqrt{5}\right)$ be rational

Let $\sqrt{3}+\sqrt{5}=a$, where a is rational.

$\therefore\ \sqrt{3}=a-\sqrt{5}$

Squaring both sides, we get

$$3=\left(a-\sqrt{5}\right)^2=a^2+5-2a\sqrt{5}$$

$$\Rightarrow\quad a^2+2-2a\sqrt{5}=0$$

$$\therefore\quad \sqrt{5}=\frac{a^2+2}{2a}\ \text{-----}(5)$$

But, $\frac{a^2+2}{2a}$ is a rational number.

Thus from (5), $\sqrt{5}$ is rational.

This contradicts the fact that $\sqrt{5}$ is irrational.

Since, the contradiction arises by assuming $\left(\sqrt{3}+\sqrt{5}\right)$ is rational.

Hence $\left(\sqrt{3}+\sqrt{5}\right)$ is irrational.

Questions 4:

Let us rewrite $\frac{1}{\sqrt{3}}$ as follows:

$$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{1}{3}\sqrt{3} ----(1)$$

If possible, let $\frac{1}{\sqrt{3}}$ be rational

Then, from (1) it follows that $\frac{1}{3}\sqrt{3}$ is rational.

Let $\frac{1}{3}\sqrt{3} = \frac{a}{b}$ where a and b are non-zero integers having no common factor other than 1.

Now, $\frac{1}{3}\sqrt{3} = \frac{a}{b} \Rightarrow \sqrt{3} = \frac{3a}{b}$ ------ (2)

But 3a and b are non-zero integers

$\therefore \frac{3a}{b}$ is rational.

Thus, from (2), it follows that $\sqrt{3}$ is rational.

This contradicts the fact that $\sqrt{3}$ is irrational

The contradiction arises by assumed that $\frac{1}{\sqrt{3}}$ is rational.

Hence $\frac{1}{\sqrt{3}}$ is irrational.

Questions 5:

(i) Consider the irrational numbers $2+\sqrt{3}$ are $2-\sqrt{3}$.

Their sum $= \left(2+\sqrt{3}\right) + \left(2-\sqrt{3}\right) = 4 =$ Rational

(ii) Consider the irrational numbers $2+\sqrt{3}$ and $2-\sqrt{3}$.

Their product $= \left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)$

$= 4 - 3 = 1 =$ Rational.

Questions 6:

(i) The sum of two rationals is always rational – True

(ii) The product of two rationals is always rational – True

(iii) The sum of two irrationals is an irrational – False

(iv) The product of two irrationals is an irrational – False

(v) The sum of a rational and an irrational is irrational – True

(vi) The product of a rational and an irrational is irrational – True